$$\frac{2r^3}{a\beta\gamma}+\left(\frac{1}{a^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)r^2=1, \text{ or } r^3+\tfrac{1}{2}\left(\frac{a\beta}{\delta}+\frac{\beta\gamma}{a}+\frac{\gamma a}{\beta}\right)r^2=\tfrac{1}{2}a\beta\gamma.$$

Example. Let $a=275$, $\beta=325$, $\gamma=429$; then $r^3+539\frac{1}{6}r^2-19170937\frac{1}{2}=0$; whence $(r-165)(r^2+704\frac{1}{6}r+116187\frac{1}{2})=0$.

$\therefore r=165$. Also, $r=-137.97+$ and $r=842.13+$.

CALCULUS.

140. Proposed by C. C. BEBOUT, Professor of Mathematics, Elgin High School, Elgin, Ill.

A pole two inches in diameter is set vertically in a level plat of ground. At a point ten feet from the ground a string is attached. A man holds the other end of the string and walks about the pole keeping the string stretched taut, and his hand at a constant distance of four feet from the ground, till the string is all wound upon the pole. If string is ten feet long, how far has his hand moved in the operation?

Solution by G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let n be the number of times the string 10 feet feet in length will encircle the cylinder in a height of 6 feet. Circumference of cylinder$=\frac{1}{6}\pi$ feet, $x=$distance between successive portions of the string on the same generating line.

Then $\sqrt{[(\frac{1}{6}\pi)^2+x^2]}=10/n$ also $nx=6$.

$$\therefore n\sqrt{\left(\frac{\pi}{6}\right)^2+\left(\frac{6}{n}\right)^2}=10 \text{ or } \frac{\pi^2n^2}{36}=64. \quad \therefore n=\frac{48}{\pi},\ x=\tfrac{1}{8}\pi.$$

The distance moved by the hand is given on page 319, No. 9, Vol. I, of the MONTHLY, and is there worked out in full. It is $s=2\pi^2r^2m^2$ where r is the radius of the cylinder and m is the number of times the string is wound around the cylinder or the same as the value of n above. $r=\frac{1}{12}$ feet, $m=n=48/\pi$.

$\therefore s=32$ feet.

Also solved by T. T. DAVIS.

141. Proposed by B. F. FINKEL, A.M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

The curve $r^n=a^n\sin n\theta$ rolls along a straight line. Show that the intrinsic equation to the evolute of the locus of the pole is $s^n=a^m[1+1/n]^n\sin\psi$. [Edward's *Differential Calculus*, page 502.

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

The distance from the point of tangency to the pole is normal to the path of the pole. Let this distance, which is a radius vector of the rolling curve, be r, and the perpendicular from the pole upon the fixed line be p.

Taking the fixed line as the axis of x, p is an ordinate of the pole, and r is the length of normal and is expressed by